

# Decay of Metastable Vacuum in Liouville Gravity

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## Abstract

A decay of weakly metastable phase coupled to two-dimensional Liouville gravity is considered in the semiclassical approximation. The process is governed by the “critical swelling”, where the droplet fluctuation favors a gravitational inflation inside the region of lower energy phase. This geometrical effect modifies the standard exponential suppression of the decay rate, substituting it with a power one, with the exponent becoming very large in the semiclassical regime. This result is compared with the power-like behavior of the discontinuity in the specific energy of the dynamical lattice Ising model. The last problem is far from being semiclassical, and the corresponding exponent was found to be  $3/2$ . This exponent is expected to govern any gravitational decay into a vacuum without massless excitations. We conjecture also an exact relation between the exponent in this power-law suppression and the central charge of the stable phase.

**Preliminaries.** Nucleation mechanism in the first order phase transitions is relatively well understood. Nucleation is the main channel of a metastable phase decay in the case of

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weak metastability (see e.g., [1]). The decay rate turns out to be *exponentially* small in the energy gap between the metastable and stable phases. The leading exponential estimate is essentially simple, controlled by the energy of the “critical droplet”. It will be schematically repeated (in the particular case of two-dimensional space) in the next section. More sophisticated calculation allows for evaluation of the pre-exponential factor as well [2]. Somewhat later similar mechanism has been studied in [3] (and then independently in [4]) in the context of quantum field theory, the thermal fluctuations responsible for the formation of critical droplet being substituted now by the quantum tunneling. The process is commonly treated dynamically in this case, the role of the critical droplet being played by the “materialization of a bubble of true vacuum” [4]. In the euclidean version the critical droplet is a kind of “instanton”, in the sense that it is a localized solution of the classical equations of motion (being however an unstable extremum of the action).

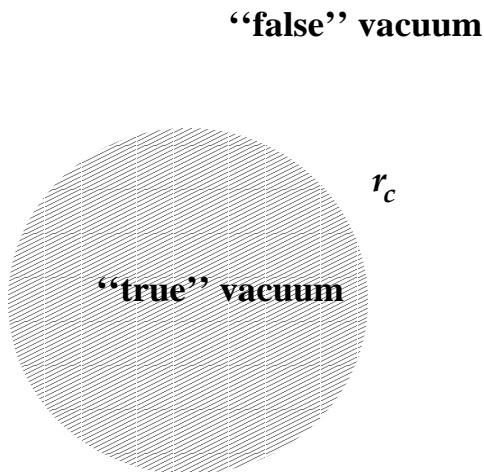


Figure 1: A critical droplet fluctuation inside the spatially infinite metastable phase

Another facet of this phenomenon can be seen when looking at the specific free energy as an analytic function of the energy gap  $\mu$  between the phases (e.g.,  $\mu$  is proportional to the magnetic field strength  $h$  in the case of low temperature phase ferromagnet in a weak field). When continued to the values corresponding to the metastable state, the free energy exhibits a branch cut with pure imaginary discontinuity, directly traceable to the “critical droplet” fluctuations (in particular, the discontinuity is exponentially small in the absolute value of  $\mu$ , as in the Eq.(5) below) [5–7]. It is essential that the singularity appears only in the thermodynamic limit, when the size of the system is sent to infinity. In the field-theoretic context the above specific free energy is interpreted as the vacuum en-

ergy density, and the presence of the imaginary part simply means that the “false vacuum” is the quantum-mechanically metastable state, thus suggesting direct interpretation of the above discontinuity in terms of the decay probability [3, 4]. In the context of the thermal phase transition, a close qualitative correlation between the discontinuity of the free energy and the rate of decay of the metastable phase is widely assumed, while exact relation has been established in the limit of weak metastability only [2].

The subject attracts much attention. Whereas the stat-mechanical aspects are of obvious interest in a wide range of disciplines from chemistry and metallurgy to astrophysics, the interest in the “false vacuum” decay is mainly due to the natural concern about the fate of

our Universe, if it faces the vacuum metastability disaster [8]<sup>3</sup>. There are even more worries after Coleman and De Luccia had studied the effects of gravity on this decay [9]. Although the Einstein gravity hardly influences the decay rate, the “realistic” parameters taken into account, the vacuum formed “a microsecond after” hardly can comfort human beings. For more similar horrors and related ethical problems see [10], and if even that is not enough, also [11]. Another interesting, although less dramatic, are the implications in the cosmology, the scenario where the whole drama is well in the past [12].

Turn to less apocalyptic matters. It has been observed recently that the effect of quantum gravity in two dimensions changes qualitatively the character of the discontinuity in the specific free energy [13]. A lattice version of quantum gravity coupled to Ising spins, known as the dynamical lattice Ising model, is exactly solvable [14] through the matrix model technique. The vacuum energy as a function of the temperature and magnetic field allows analytic expression. In the case of ordered phase, at small absolute values of the magnetic field there is indeed a branch cut, the discontinuity being suppressed as a *power* rather than exponentially.

Some details of this observation are briefly recapitulated in one of the subsequent sections. It suggests naturally that this power law also governs the decay rate in the limit of weak metastability. We find it therefore instructive to repeat the semiclassical critical droplet calculation taking into account relevant effects of the quantum Liouville gravity. The latter, rather than the Einstein gravity, governs the dynamics of the geometry in two dimensions [15]. The Liouville gravity is characterized by a dimensionless coupling constant, related to the central charge  $c$  of the massless modes around the vacuum. The semiclassical limit corresponds to large negative  $c \rightarrow -\infty$ . In this limit we find that the decay is controlled by a gravitational version of critical droplet, which we call the “critical swelling”. This localized distortion of the metric is induced by the nucleus of the “true” vacuum. It looks more like a “cap” at the extreme semiclassical limit  $c\mu \rightarrow -\infty$  and like a “bubble” in the opposite weak metastability limit  $c\mu \rightarrow -0$ . In the last case the decay rate behaves as  $\mu^{-c/6}$  at  $\mu \rightarrow 0$ , the *power* law replacing the exponential suppression in the flat.

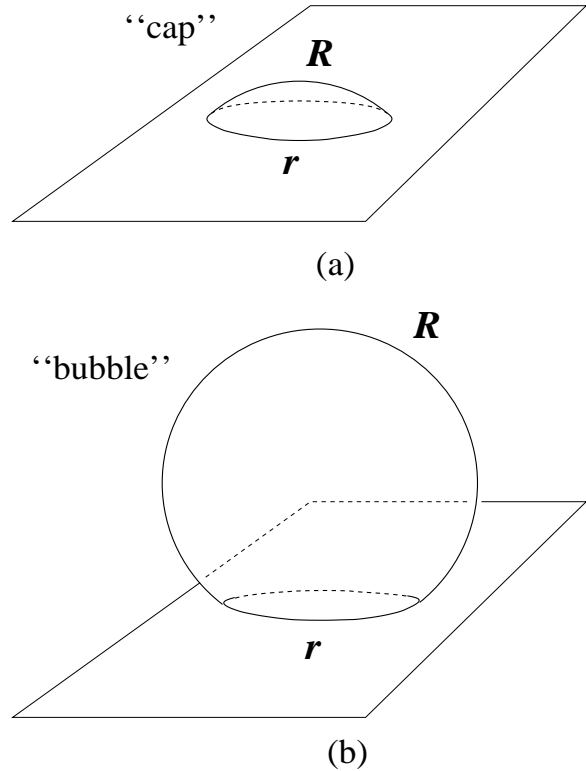


Figure 2: “Critical swellings in the case  $s < 1$  (a) and  $s > 1$  (b).

<sup>3</sup>sometimes seen as “the ultimate ecological catastrophe”, see [9] for the developed concept.

This result is semiclassical. It is natural to expect in general some *power* behavior

$$P \sim \mu^\delta, \quad (1)$$

with the parameter  $\delta$  depending on the central charge  $c$  of the stable vacuum. An interesting question arises about the exact dependence of this exponent on  $c$ . From the exact solution one can figure out that  $\delta = 3/2$  for the decay into a vacuum without massless modes, where  $c = 0$ . At the end of this paper we develop arguments in favor of the following exact formula

$$\delta = \frac{13 - c + \sqrt{(1 - c)(25 - c)}}{12} \quad (2)$$

which gives  $3/2$  at  $c = 0$  while  $\delta \sim (13 - c)/6 + O(1/c)$  in the semiclassical limit.

**Nucleation in flat background.** To begin with we recall the standard consideration about the decay of a metastable state in statistical mechanics, following [1], §162. Formally identical analysis applies to the semiclassical tunneling decay of the “false vacuum”. Here we restrict the discussion to the leading exponential factor, although more accurate treatment, which allows to estimate the pre-exponential multiplier too, can be carried out, see [3, 4]. In [16] the two-dimensional version is specially treated in very details.

Consider a flat infinite two-dimensional plane filled up with a metastable vacuum<sup>4</sup>. Its specific energy  $\mathcal{E}_1$  is meant to be slightly above the energy of the “true” stable vacuum  $\mathcal{E}_0$ . We denote  $\mu$  the (positive) difference between the two energies, i.e.,  $\mathcal{E}_1 = \mathcal{E}_0 + \mu$ . Imagine now a fluctuation inside this false sea, where a region  $\Gamma$  of true vacuum is formed, and let  $A(\Gamma)$  and  $p(\Gamma)$  be respectively the area and perimeter of  $\Gamma$ . Also, let  $\sigma > 0$  be the surface tension at the border between true and the false vacuum. The energy of the fluctuation  $\Delta E(\Gamma) = p(\Gamma)\sigma - A(\Gamma)\mu$  is a result of a competition between the gain in the bulk and the expense at the perimeter. Weak metastability means that the characteristic  $\Delta E$  is much smaller than the temperature (in the field theoretic context  $\Delta E$  is interpreted as the action of the “instanton” fluctuation and is compared with the Plank constant). In this case the estimate for the probability  $P(\Gamma)$  of this fluctuation is

$$P(\Gamma) \sim \exp\left(-\frac{\Delta E(\Gamma)}{T(\hbar)}\right) \quad (3)$$

where  $T$  (or the Plank’s constant  $\hbar$  in the case of quantum tunneling) is the parameter governing the fluctuations. In what follows we always imply that the energy (action) is measured in the units of  $T(\hbar)$  and omit the corresponding factors.

By usual arguments [3, 4], the specific (per unit area of the 2D space) probability of the false vacuum decay is controlled mainly by the configuration which extremizes (3), the decay instanton, a.k.a. the critical droplet. The instanton is a circle of radius

$$r_c = \sigma/\mu \quad (4)$$

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<sup>4</sup>Considerations of this paragraph go in a space of any dimensionality as well. Here we take it two-dimensional to match with what follows.

(fig.1). In this calculation we assume that the size of the critical droplet is large as compared to all microscopic scales, such as the thickness of the border of the droplet etc. This is certainly the case in the weak metastability limit  $\mu \rightarrow 0$  (as the eq.(4) indicates). Evaluating the energy of the critical droplet one finds  $\Delta E_c = \pi\sigma^2/\mu$ . This means that at small  $\mu$  the decay rate is exponentially suppressed

$$P \sim \exp\left(-\frac{\pi\sigma^2}{\mu}\right) \quad (5)$$

in the energy gap  $\mu$ .

**Nucleation in 2D Liouville gravity.** Now we're going to couple our system to the gravitating background, where the effective action of the gravity is given by the Liouville action  $A_L$  [15]. Choosing the coordinate system where  $g_{ab} = e^\varphi \delta_{ab}$  we have

$$A_L = \frac{k}{96\pi} \int (\partial_a \varphi)^2 d^2x \quad (6)$$

Parameter  $k$  is related to the effective number of massless modes in the corresponding vacuum, called the central charge  $c$  [17]. In the semiclassical limit  $c \rightarrow -\infty$  it has been found that  $k \sim -c$  [15]. In general the relation between  $k$  and  $c$  depends on the regularization prescription. Regularization with respect to a fixed background metric implies the following expression [18]

$$k = 6b^{-2} \quad (7)$$

in terms of the standard Liouville parameter  $b$  related to  $c$  through

$$26 - c = 1 + 6(b^{-1} + b)^2 \quad (8)$$

As in the case of the flat background, here we are interested in the specific (per unit area) probability of the decay in the limit when the area is very large <sup>5</sup>. Correspondingly, our setting of the decay problem is as follows: at the beginning we have an infinite, globally flat space filled with the metastable phase. Semiclassically this is described by the solution  $\varphi = \text{const}$ , which we choose to be  $\varphi = 0$ . Imagine a local fluctuation somewhere near the origin, so that  $\varphi = 0$  for  $|x| > r$  with some  $r$ . At  $|x| < r$  we have a seed of the stable phase, which has negative energy density  $-\mu$  with respect to that of the false one. Inside the droplet we are looking for the extremum of the functional

$$A_L = \int_{|x| < r} \left( \frac{k}{96\pi} (\partial_a \varphi)^2 - \mu e^\varphi \right) d^2x \quad (9)$$

with respect to the scale function  $\varphi(x)$  at  $|x| < r$ . The extremum is of course a solution to the positive curvature Liouville equation

$$\partial\bar{\partial}\varphi = -12\pi\mu k^{-1}e^\varphi \quad (10)$$

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<sup>5</sup>This limit can be achieved by fine-tuning the “bare” cosmological constant  $\lambda_0$  to a certain critical value  $\lambda_c$ , so that the physical cosmological constant vanishes. That is how it is done in the matrix models of gravity (see e.g. example in the next section). This critical value (more precisely,  $-\lambda_c$ ) is interpreted as the intensive part of the specific free energy, and it is this quantity which develops the singularity  $\mu^\delta$  at  $\mu \rightarrow 0$ .

The general solution respecting the symmetry of the problem is a sphere

$$e^\varphi = \frac{4R^2a^2}{(1 + a^2x\bar{x})^2} \quad (11)$$

of radius  $R = R_c$  with

$$R_c^2 = \frac{k}{24\pi\mu} \quad (12)$$

and a scale parameter  $a$  to be determined from the continuity at the boundary of the swelling. It will turn instructive not to imply the relation  $R = R_c$  from the very beginning considering  $\mu$  and  $R$  as unrelated parameters, to be bound later in the extremum condition.

We need to sew this solution with the outside region in a continuous way, i.e., at  $|x| = r$  we have  $2Ra = 1 + a^2r^2$ . This quadric has two solutions for  $a$

$$a_\pm = \frac{R \pm \sqrt{R^2 - r^2}}{r^2} \quad (13)$$

Here  $a_+$  corresponds to the “bubble” solution, while  $a_-$  is a “cap” one, see fig.2. Both are conveniently uniformized through the parameterization

$$2R = r(t + t^{-1}) \quad (14)$$

In this way both branches are covered if we set  $ar = t$  and let  $t$  run from 0 to  $\infty$ ,  $t < 1$  corresponding to the “cap” configuration and  $t > 1$  being related to the “bubble”. It is easy to evaluate in this parameterization both terms of the Liouville action (9)

$$\begin{aligned} \frac{k}{24\pi} \int \partial\varphi \bar{\partial}\varphi d^2x &= \frac{k}{6} \left( \log(1 + t^2) - \frac{t^2}{1 + t^2} \right) \\ \mu \int_{|x| < r} e^\varphi d^2x &= 4\pi\mu R^2 \frac{t^2}{1 + t^2} \end{aligned} \quad (15)$$

Adding the surface tension term  $2\pi\sigma r$  we find the total energy of the swelling to be varied

$$E = \frac{k}{6} \left( \log(1 + t^2) - \frac{t^2}{1 + t^2} - x^2(1 + t^2) + 2sx \right) \quad (16)$$

where we have used the notation

$$s = \sigma \sqrt{\frac{6\pi}{k\mu}} \quad (17)$$

instead of the surface tension. Also we introduced another parameter

$$2x = r/R_c. \quad (18)$$

The relevant extremum is achieved at  $t = s$  and  $x = s/(1 + s^2)$ . The extremal value

$$E_c = \frac{k}{6} \log(1 + s^2) \quad (19)$$

It can be verified directly that in both cases the chosen extremum corresponds to a saddle point in the parameters  $(R, r)$ , in the case  $s \ll 1$  the unstable direction being mostly along the “perimeter” direction  $r$ , while in the bubble case  $s > 1$  it is generally along the “inflation” direction  $R$ . Finally, we find for the decay rate

$$P \sim \left(1 + \frac{6\pi\sigma^2}{k\mu}\right)^{-k/6} \quad (20)$$

At the extreme limit  $k \rightarrow \infty$  we are back to the classic droplet result (5). On the contrary, the weak metastability limit  $\mu \rightarrow 0$  is characterized by a power-like suppression  $P \sim (k\mu/\sigma^2)^{k/6}$ .

**Gravitational Ising vacuum energy.** We recall in this section very briefly the dynamical lattice Ising model (DLIM), as it has been proposed in [14], and present without derivation the form of its exact solution in the scaling limit near the ferromagnetic phase transition. The formulation is standard for the two dimensional random lattice models and starts with an ensemble  $\{G_N^{(g)}\}$  of planar graphs (in general of genus  $g$ ) and of size  $N$ . It is considered well established that the scaling singularity, which is supposed to correspond to a continuous limit, doesn’t depend on the details of this ensemble (neither on most microscopic details of the spin model located on the graph). Here for definiteness we will imply a very particular realization of  $\{G_N^{(g)}\}$  as the graphs made exclusively of triangles, the triangulations, like an example in fig.3. In this case the size  $N$  can be given precise meaning, the number of triangles in  $G_N^{(g)}$ .

In DLIM we associate a spin variable  $\sigma_i = \pm 1$  with each triangle  $i = 1, 2, \dots, N$ . As in the usual Ising, only “nearest neighbor” triangles  $\langle ij \rangle$  (i.e., having a common edge) contribute to spin-spin interaction

$$\mathcal{H}[\{\sigma\}] = \sum_{\langle ij \rangle} K \sigma_i \sigma_j + \sum_i H \sigma_i \quad (21)$$

where  $K$  is the “thermal” parameter (called traditionally the exchange integral) and  $H$  is the external magnetic field. In this study we consider the large  $N$  limit characteristics, namely the specific free energy (per triangle) of DLIM. This energy is an intensive characteristic and as such doesn’t depend of global properties, like  $g$ . For our purposes it is most convenient to

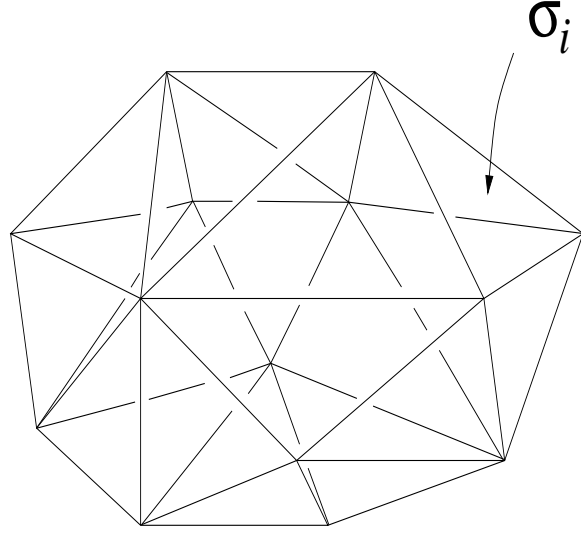


Figure 3: An example of triangulation of a sphere  $g = 0$ . Ising spins are attached to the triangles.

start with the spherical partition function ( $g = 0$ ), where the result of [14] applies directly. Introduce the microcanonic spherical partition function

$$Z_N(K, H) = \sum_{\{G_N^{(0)}\}} \sum_{\{\sigma\}} \exp(-\mathcal{H}[\{\sigma\}]) \quad (22)$$

Here, in addition to the standard Ising sum over the spin configurations  $\{\sigma\}$  the sum over  $\{G_N^{(0)}\}$  is also implied, the last being the lattice version of the quantum gravity path integral over the metrics. Considering the leading logarithmic behavior at large  $N$

$$\log Z_N(K, H) \sim -N\mathcal{E}(K, H) \quad (23)$$

we readily see that the intensive energy  $\mathcal{E}(K, H)$  is determined by the position of the right-most singularity in the grand partition function

$$Z(K, H, X) = \sum_N e^{-XN} Z_N(K, H) \quad (24)$$

in the “chemical potential”  $X$ .

Let  $(X, K, H) = (X_c, K_c, 0)$  be the position of the double singularity in the grand partition function, caused by the combined divergence in size  $N$  in (24) and of (22) due to the long correlations of spins at the ferromagnetic phase transition. The singular part  $Z_{\text{sing}}(x, t, h)$  of the partition function depends in a scaling way on the deviations  $x \sim X - X_c$ ,  $t \sim K_c - K$  and  $h \sim H$  (precise definitions depend on the choice of scale). Boulatov and Kazakov [14] showed that the function

$$u = \partial^2 Z_{\text{sing}} / \partial x^2 \quad (25)$$

solves the following simple algebraic equation

$$x = u^3 + \frac{3}{2}tu^2 + \frac{h^2}{(u+t)^2} \quad (26)$$

As usual the continuous theory deals with the *singular* part of the intensive energy, which, up to sign coincides with  $x_c$ , the singularity of equation (26). It is easy to see [13] that the relevant singularity is located at

$$x_c = \frac{t^3}{2} f(5f^2 + 9f + 3) \quad (27)$$

where  $f$  is an appropriate solution to the following fifth order algebraic equation

$$f(f+1)^4 = \frac{2h^2}{3t^5} \quad (28)$$

As it is argued in ref. [13], the low temperature phase  $t < 0$  of DLIM corresponds to the solution

$$f = -1 + \sum_{n=1}^{\infty} \frac{\Gamma(5n/4 - 1)}{n! \Gamma(n/4)} \left( \frac{2^{1/4} h^{1/2}}{3^{1/4} (-t)^{5/4}} \right)^n \quad (29)$$



and leads to the following singularity at  $h \rightarrow 0$

$$\begin{aligned}
t^{-3}x_c &= \frac{1}{2} + 3 \sum_{n=2}^{\infty} \frac{(n-1)\Gamma(5n/4-3)}{n!\Gamma(n/4)} \left( \frac{2^{1/4}h^{1/2}}{3^{1/4}(-t)^{5/4}} \right)^n \\
&= \frac{1}{2} - \frac{\sqrt{6}h}{(-t)^{5/2}} + \frac{2^{3/4}h^{3/2}}{3^{3/4}(-t)^{15/4}} + \frac{h^2}{4(-t)^5} + \dots
\end{aligned} \tag{30}$$

The free energy exhibits the square root branch cut at  $h = 0$  with the exponent  $3/2$ .

**Discussion and Speculations.** There are serious reasons to believe that in two dimensions the Liouville gravity affects the standard nucleation mechanism of the metastable decay in a significant way. The standard exponential suppression of the decay rate is substituted by a weaker power-like one, changing qualitatively the picture. We were only able to obtain more precise information either in the semiclassical limit, controlled by the instanton, or in the exactly solvable dynamical lattice Ising model. One could expect in principle a one-parameter family of universality classes, depending on the central charge in the final state of the decay. A better understanding of this family is desired. In this relation different steps would be valuable. One thing, which is not attempted in the present study, is the one-loop (and higher) corrections to the critical swelling instanton. Such calculation would provide a constant term in the semiclassical expansion  $\delta \simeq -c/6 + \text{const}$  of the exponent in (1), and thus support (or disprove) the conjectured exact form (2). It is also useful to look for more solvable examples among the dynamical lattice statistical systems exhibiting metastability and, preferably, having non-trivial massless modes in the stable vacuum. And of course, metastability opens a large playground for Monte-Carlo simulations, essentially dynamical processes. Many appropriate models are known (e.g., the “generalized random lattice Ising model of ref. [19]), which certainly can be put to metastable state, while the central charge in the stable vacuum remains completely under our control.

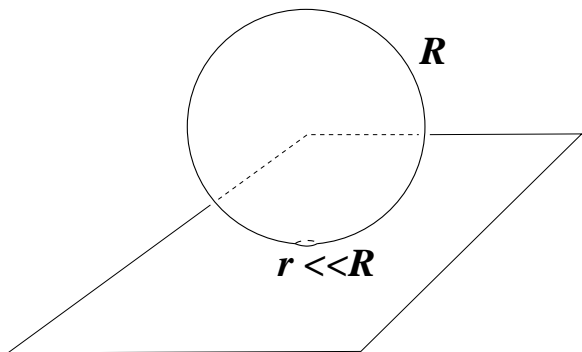


Figure 4: Critical swelling in the regime of extremely weak metastability  $\mu \rightarrow 0$ .

The Eq.(1) applies to the decay probability in the situation when the metastable phase is accommodated by infinite globally flat geometry, or more generally when all the length scales associated with that geometry are large as compared with  $\mu^{-1}$ . In particular, the physical cosmological constant of the metastable phase must be much smaller than  $\mu$ . This settlement is perfectly appropriate for the situation of the previous sections, where we were interested in the intensive part of the specific free energy.

But the semiclassical calculations of the third section certainly can be generalized to incorporate the effect of the cosmological constant in an interesting way.

To conclude, we speculate a conjecture for the exact relation (2) between the central charge and the exponent in the decay law. It is easy to see, that in the limit of extremely weak metastability  $\mu \rightarrow 0$  the “bubble” configuration of fig. 2(b) tends to an almost complete sphere, connected to the metastable plane through a small “throat”. The last can be seen as a pointlike puncture (fig.4). We recall in this context that the partition function of a compact sphere behaves as  $\mu^{b^{-2}+1}$  where  $b$  is from (8). This is an exact result of quantum Liouville field theory, which can be found e.g., in [20]. A puncture corresponds to a differentiation with respect to  $\mu$  and the action of the limiting swelling is evaluated as  $\mu^{1/b^2}$ , i.e., the power law conjectured in (2). In this consideration we assume (with no really good reasons) that the essentially semiclassical arguments about the critical swelling, which we used to reduce the problem to a punctured sphere, apply qualitatively also at general quantum  $c$ .

Let us mention also that the exponent equal to  $1/b^2 + 1$  (which differs by 1 from (2)) was proposed recently by A.Polyakov [21] in apparently different context, in relation with possible unstability of the de Sitter space. Obviously, understanding precise relation to the decay problem considered here would be of much interest.

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